VI Workshop de Teses e Dissertações em Matemática

Caderno de Resumos

20 a 22 de junho de 2016
ICMC-USP

Realização:

Apoio:

Objetivo

Em sua sexta edição, o Workshop de Teses e Dissertações em Matemática tem como finalidade promover a integração e a divulgação da pesquisa do programa de pós-graduação em matemática do ICMC-USP. Consiste em um momento propício para discussões, uma vez que os alunos em fase final da produção de sua tese ou dissertação, são incentivados a ministrarem palestras divulgando os resultados obtidos em suas pesquisas.

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Palestra de encerramento

O fantasma do plágio na redação científica

Marcelo Krokoscz

O objetivo principal desta palestra será refletir sobre o plágio no campo acadêmico a partir de um referencial teórico selecionado no âmbito nacional e internacional. Especificamente, pretende-se conscientizar e capacitar os participantes para compreenderem o assunto na sua complexidade e profundidade, de modo que, consequentemente, desenvolvam uma visão esclarecida e uma postura preventiva em relação ao plágio no campo da pesquisa.

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Palestras

Existence of free actions, cohomology ring of orbit spaces and applications

Ana Maria Mathias Morita and Denise de Mattos

Let G be a topological group and X a topological space. There is a natural question associated with the pair (G, X) about existence of free and continuous actions of G on X. If such an action exists, other natural question is the study of properties of the orbit space X/G and in this setting, we have the problem of computing the cohomology ring of X/G. Our purpose here is to determine which Dold manifolds $P(m, n)$ admit fixed points free involutions (or $\mathbb{Z}_2$-free actions), and in the affirmative cases, to compute the cohomology ring of the corresponding orbit spaces. A Dold manifold $P(m, n)$ is the quotient space of $\mathbb{S}^m \times \mathbb{C}P^n$ by product involution $A \times C$, where A is the antipodal and C is the complex conjugation. The main tool to be used to approach the problem is the Leray-Serre spectral sequence associated with Borel fibration.

Acknowledgements: We would like to thanks CAPES by financial support.

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Some spaces of special elliptic n-gons

Felipe A. Franco and Carlos H. Grossi

In [Ana], relations between reflections in (positive or negative) points in the complex hyperbolic plane are studied. Any relation between reflections gives rise to a representation $H_n \to \mathrm{PU}(2,1)$, where $H_n = \langle r_1, \dots, r_n \mid r_n \dots r_1 = 1, r_i^2 = 1\rangle$ is the hyperelliptic group and $\mathrm{PU}(2,1)$ is the group of complex automorphisms of the holomorphic 2-ball. Such representations, when discrete, may lead to important examples of complex hyperbolic manifolds.

We study relations between *special elliptic isometries* in the complex hyperbolic plane. A special elliptic isometry can be seen as a 'rotation' around a fixed axis (a complex geodesic, including a positive one). Reflections in points are a particular case. A special elliptic isometry $R_\alpha(p)$ is determined by specifying a nonisotropic point p (the polar point to the fixed axis) and a unitary complex number α, the *angle* of the isometry.

We call a relation $R_{\alpha_n}(p_n) \dots R_{\alpha_1}(p_1) = 1$ in $\mathrm{PU}(2,1)$ a (special elliptic) *n-gon*. Up to a few simple discrete invariants, we expect the (generic part of the) spaces of 4 and 5-gons with fixed angles to be connected by means of *bendings*: the *i-bending* is the change, in the relation $R_{\alpha_n}(p_n) \dots R_{\alpha_1}(p_1) = 1$, of $R_{\alpha_{i+1}}(p_{i+1})R_{\alpha_i}(p_i)$ by $(R_{\alpha_{i+1}}(p_{i+1}))^c(R_{\alpha_i}(p_i))^c$, where c runs over the centralizer of $R_{\alpha_{i+1}}(p_{i+1})R_{\alpha_i}(p_i)$. It is not difficult to see that every i-bending preserves an n-gon with fixed angles. The case of 4 and 5-gons without the angle restriction requires (at least) a new type of bending which we call an *f-bending*.

Hopefully, the detailed study of the space of special elliptic n-gons may shed some light on the theory (still in its infancy) of complex hyperbolic Teichmüller spaces; in principle, it could also constitute a tool to be applied to the particular and very important case of $\mathcal{T}H_n$.

Acknowledgements: We would like to thanks FAPESP for the financial support.

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Asymptotic behavior of Neutral Functional Differential Equations

Fernando Gomes de Andrade and Miguel Vinícius Santini Frasson

In the study of a class of Neutral Functional Differential Equations (NFDE) it is known that the asymptotic behaviour of solutions is obtained from the spectral properties of infinitesimal generator A of the solution semigroup. Furthermore, if exists a dominant eigenvalue of A, then we can describe the asymptotic behaviour of solution using the spectral projection. In the general case, however, perhaps there is no such eigenvalue and the spectral projection results do not apply. The aim is to use techniques involving Laplace Transform and overcome the non existence of dominant eigenvalue.

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Invariance Principle and Frame Flow

Joás Rocha, Régis Varão and Ali Tahzibi

Inspired by the Avila- Viana-Wilkinson's paper [1], where they obtain a complete description of a C^1-neighborhood volume preserving of time one map of a geodesic flow on a closed surface with negative curvature, we propose study a neighborhood of the natural generalization of that kind of map: the time one map of a frame flow.

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Algebraic restriction to non quasi homogeneous curves

Lito E. Bocanegra R. and Roberta Godoi Wik Atique

We study the algebraic restriction on a non quasi homogeneous curves in the symplectic space $\mathbb{C}^2$ and the possible solution of a symplectic classification problem to this non quasi homogeneous curves following the same idea of [1].

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Topology of Real Analytic Singularities

Maico Felipe Silva Ribeiro and Raimundo N. Araújo dos Santos

It is well known that, given a real analytic map germ $f : (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$, $2 \leqslant p < n$, under the Milnor's conditions (a) and (b), see [Ma], the fibration in the called Milnor tube exists. In addition, if we assume that the Milnor set of (the canonical) projection $M(\dfrac{f}{\|f\|})$ is empty as a germ of set, then it is the projection of a fibration on the complement of the link on sphere to sphere. This is called the Milnor fibration on spheres. See [ACT, AT2, JJJ, RSV, Mi] for details.

In this project we will be developing tools and techniques that allow us to study in a first step the equivalence between these two fibrations. Moreover, we would like to study the topology of the fibers and links of maps under these two conditions, searching for topological and algebraic invariants of singularities (see for instance [ADD]) which allow us to start a classification of these mathematical objects. For this, we will be working on several articles published recently on this issue trying to find a more general theory which can unify all main results.

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Equivariant binary differential equations

Patrícia Tempesta and Miriam Manoel

Symmetry is a natural property in many mathematics models and due to its large occurrence in dynamical systems, it has been studied by several authors in the last decade. This study is motivated by the recognition of symmetry in binary differential equations (BDE's), that are implicit differential equations of the form

$$a(x, y)dy^2 + b(x, y)dxdy + c(x, y)dx^2,$$

where a, b, c are smooth real function defined on some open set of $\mathbb{R}^2$. In this talk we present the definition of symmetry for a binary differential equation via group representation theory and show the algebraic formulae that allow us recognize the symmetry group associated with a BDE looking only for its configuration. We describe the structure of symmetry groups of the special class of linear BDEs, namely when a, b and c are linear functions. Also, through an analysis of the number of invariant lines in the configuration associated with a BDE, we present a classification of these groups. We illustrate with examples.

Acknowledgements: We would like to thanks CAPES by financial support.

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Attractors for a class of nonlinear viscoelastic equations with memory

Paulo N. Seminario Huertas and T. F. Ma

This paper is concerned with a nonlinear viscoelastic equation with past history,

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^\infty \mu(s)\Delta u(t-s)\,ds + f(u) = h,$$

defined in a bounded domain of $\mathbb{R}^3$ and with $0 \leq \rho < 4$. This class of equations was studied by many authors but well-posedness and long-time dynamics were only established recently. The existence of global attractors was considered with additional viscous or frictional damping. In the present work we show that existence of global attractors can be established with the sole weak dissipation given by the memory term. This is done by defining a new multiplier. In addition, with respect to the strongly damped problem, we also discuss the upper semicontinuity of the attractors as $\rho \to 0$.

Key words: Viscoelastic equation, memory, nonlinear density, global attractors, upper semicontinuity.

AMS Classification: 37B55, 35L70, 35B41.

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Operads

Pryscilla dos S. F. Silva and Igor Mencattini

Operads are mathematical devices which describe many kinds of algebras (associative, commutative, Lie, Poisson, A-infinity, etc.) from a conceptual point of view. So they are a good tool for studying algebraic structures and obtaining relationships between them . In order to highlight this use , we present the definition of operads, some examples and applications .

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Strict positive definiteness of multi-level covariance functions on compact two-point homogeneous spaces

Rafaela N. Bonfim and Valdir A. Menegatto

A l-level covariance function on a compact two-point homogeneous space M is a positive definite real matrix function $(x, y) \in M \times M \to [f_{\mu\nu}(x, y)]_{\mu\nu=1,2,\ldots,l}$. The positive definiteness refers to the fact that the $ln \times ln$ block matrix $[f_{\mu\nu}(x_i, x_j)]$ is nonnegative definite whenever n is a positive integer and $x_1, x_2, \ldots, x_n$ are n distinct points on M. In this work we will provide a characterization for the continuous and isotropic l-level covariance functions on M, thus complementing a similar result on spheres usually attributed to either E. J. Hannan or A. M. Yaglom in the literature. Going one step further, we will present a necessary and sufficient condition for the strict positive definiteness of a continuous and isotropic l-level covariance function on M, in the case in which M is not a sphere. As usual, the strict positive definiteness of a positive definite l-level covariance function on M carries the following additional requirement: all the matrices $[f_{\mu\nu}(x_i, x_j)]$ mentioned above are, in fact, positive definite. Finally, in the case $l = 2$, we will present an alternative necessary and sufficient condition for strict positive definiteness based upon the diagonal elements f_{11} and f_{22} in the matrix representation for the covariance function.

Acknowledgements: We would like to thanks FAPESP by financial support.

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Disc bundles over surfaces with the geometry of the holomorphic bidisc

Sidnei Furtado Costa and Carlos Henrique Grossi Ferreira

Our main objective is to present some very simple examples of disc bundles $M \to \Sigma_g$ over closed orientable surfaces of genus $g \geqslant 1$ with geometry modeled on the product $\mathbb{D}^2 := \mathbb{D} \times \mathbb{D}$ of two Poincaré hyperbolic planes.

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Anosov actions associated with contact pairs

Uirá Norberto Matos de Almeida and Carlos Alberto Maquera Apaza

In 1992, Ives Benoist, Patrick Foulon and Francois Labourie, proved, in their celebrated paper [1] proved a classification theorem for contact Anosov flows. They proved that contact Anosov flows are "essentially" geodesic flows on negatively curved manifolds. Using the concept of contact pairs, developed by Gianluca Bande and Amine Hadjar [2], we define a higher dimension analogue of contact Anosov flows, the Anosov actions associated with contact pairs, and take steps into proving a similar classification theorem.

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Differentiability of isotropic positive definite functions

Victor Simões Barbosa and Valdir Antonio Menegatto

In this work we study continuous kernels on compact two-point homogeneous spaces which are positive definite and isotropic (zonal). Such kernels were characterized by R. Gangolli ([2]) some forty years ago and are very useful for solving scattered data interpolation problems on the spaces. In the case the space is the d-dimensional unit sphere, J. Ziegel showed in [3] that the radial part of a continuous positive definite and zonal kernel is continuously differentiable up to order $\lfloor (d-1)/2 \rfloor$ in the interior of its domain. The main issue here is to obtain a similar result for all the other compact two-point homogeneous spaces.

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Hamiltonian elliptic system in dimension two with potencials which can vanish at infinity

Yony Raúl S. Leuyacc and Sérgio H. Monari Soares

We will focus on the existence of positive radial solutions to the following Hamiltonian elliptic system

$$\begin{cases} -\Delta u + V(x)u = g(v), & x \in \mathbb{R}^2, \\ -\Delta v + V(x)v = f(u), & x \in \mathbb{R}^2, \end{cases}$$

where V is a nonnegative radial function which can vanish at infinity like $\dfrac{1}{|x|^a}$ with $0 < a < 2$ and $f, g \in \mathcal{C}(\mathbb{R})$ possess nonlinearities in the critical growth range, with respect to the Trudinger-Moser growth.

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Pôsteres

Algebraic function fields

Alex Freitas de Campos and Herivelto Martins Borges Filho

Let K be a field and let x be transcendental over K. We call F/K an algebraic function field (of one variable) over K if F is a finite field extension of $K(x)$. This kind of field extension appears in numerous branches of mathematics among which we cite algebraic geometry (the field of rational functions of an algebraic curve), the theory of compact Riemann surfaces (the meromorphic functions on a compact Riemann surface) and coding theory (the construction of codes by means of algebraic function fields). In this work we give a brief description of the general theory of algebraic function fields, in order to clarify the most important results.

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Configuration spaces for robots

César Zapata and Denise de Mattos

Configuration spaces are inspired by applications in multi-agent robotic systems [2], where a key difficulty in the design of multi-agent robotic systems is the zise and complexity of the space of possible designs. Michael Farber in [1] introduced a numerical homotopy invariant, *Topological Complexity* $TC(X)$, of a topological space X. It measures the complexity of all possible motion planning algorithms for the system.

Consider an automated factory equipped with n mobile robots, which very one move on an $C-$space X. A common goal is to place several such robots in motion simultaneously, controlled by an algorithm that either guides the robots from initial positions to final positions. These robots cannot tolerate collisions. As a first step at modelling such a system. The *configuration space* of ordered n distinct points on X is given by

$$F(X,n) := \{(x_1, \cdots, x_n) \in X^n : x_i \neq x_j \quad \textbf{if } i \neq j\}.$$

It represents those configurations of n points in X which not experience a collision. Therefore, $F(X,n)$ forms an acceptable model for robot motion planning on an unobstructed floor.

The aim of this work is to give the majority of examples of $C-$spaces which rise in engineering and the principal result know about these, specially their cohomology algebra which will be use to calculate their topological complexity.

Examples: [m n-D multi-agent robotic (n-D rigid bodies) systems which not experience a collision] The $C-$space of m n-D multi-agent robotic systems which not experience a collision that can translate in $\mathbb{R}^n$ is:

$$C - \mathit{space} = F(\mathbb{R}^n, m).$$

The $C-$space of m n-D multi-agent robotic systems which not experience a collision that can rotate in $\mathbb{R}^n$ is:

$$C - \mathit{space} = F(SO(n), m).$$

The $C-$space of m n-D multi-agent robotic systems which not experience a collision that can translate and rotate in $\mathbb{R}^n$ is:

$$C - \mathit{space} = F(\mathbb{R}^n \times SO(n), m).$$

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Connectivity of algebraic varieties affine

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In [5] John Milhor showed that given a germ of holomorphic function$f$: $(\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ exists a smooth fibration around the singularity for all spheres with suffiiciently small radius (but on the complement of the set $f^{-1}(0)$) on the sphere S^1. Futhermore, when the origin is an isolated singular point, Milnor showed that the fiber has the homotopic type of a finite bouquet of spheres $n-$dimensional. Thus showing that the fiber is strongly connected. Posteriorly, kato and Matsumoto [4] generalized the Milnor result showing that if the dimension complex of the singular set of f is s, then the fiber is $(n-1-s)-$connected. Therefore, establishing a strict relation between the dimension of the singular set and the connectivity degree of the fibration fiber. For germs of real analytic applications $f : (\mathbb{R}^m, 0) \to (\mathbb{R}^k, 0)$, $m > k \geq 2$, wit isolated singularity at the origin, Milnor also showed the existence of a smooth fibration in all sphere of radius small enough around the singularity, and since the dimension of the variety $g^{-1}(0) - \{0\}$ is positive, then the fiber is $(k-2)-$connected. In the global complex case that is for analitycal polynomial funcions $f : \mathbb{C}^{n+1} \to \mathbb{C}$ is well known there is a finite set of values $\Gamma \subset \mathbb{C}$, such values called atypical such that the restriction $f : \mathbb{C}^{n+1} \setminus f^{-1}(\Gamma) \to \mathbb{C} \setminus \Gamma$ is a soft bundle, and the global fiber has the homotopy type of a $CW-$complex finite $n-$dimensional. Futhermore, if f satisfies certain regularity conditions in the infinity, for example, if f is *tame*, then the global fiber also has the homotopic type of a finite bouquet of spheres $n-$dimensional. Some of these results were generalized in [3], For example. The objetive of this project is to develop a study of real and complex cases, local and global; which allows us to understand each situation how the components of the singular set of positive dimension interfere in the fiber connexity degree of those bundles. For this, we use the tools and techniques of Singularity Theory, Stratification Theory of the differential and algebraic topology..

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Some applications of theory sets

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Set theory currently serves for many applications of combinatorial aspect in several areas of mathematics. Results in functional analysis and general topology often have some combinatorial aspect in their statements, but others, though no combinatorial aspect in their statements, have demonstrations that use such techniques. In this work we describe some translations of certain spaces to the language of the set theory and combinatories. The first result classifies the countable Hausdorff compact topological spaces K, in the sense of saying that they are homeomorphic to an ordinal with the order topology. The second result affirms the existence of an independent family of cardinality of the continuum in an infinite Boolean Algebra A, and the relationship of the property Grothendieck in the space $C(s(A))$ (where $s(A)$ is the space Stone of A, i.e. the compact Hausdorff space such that the set of all clopen sets of $s(A)$ is homeomorphic to A) with own algebra A.

Acknowledgements: We would like to thanks CAPES by financial support.

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De Rham Theorem

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The influence of De Rham theorem, proved in 1931, was particularly great during the development of Hodge theory and sheaf theory. In simplest terms, the theorem says that the De Rham cohomology is a topological invariant. In an abstract topological spaces, the various different ways of defining holes are described by the various definitions of the cohomology of a manifold, and the De Rham theorem says that all of these methods of measuring holes are the same. Precisely the theorem affirm that if M is smooth manifold, then the de Rham homomorphism

$$k_p^* : H_{deR}^p(M) \to H_{\Delta}^p(M; R)$$

induces isomorphisms in cohomology for all p, where $H_{deR}^p(M)$ is the classical pth De Rham cohomology group for M and $H_{\Delta}^p(M; R)$ is the pth classical singular cohomology group of M.

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An Ordering for Groups of Pure Braids

Letícia Melocro and Denise de Mattos

Braid groups first appeared, although in a disguised form, in an article by Adolf Hurwitz published in 1891 and devoted to ramified coverings of surfaces. The notion of a braid was explicitly introduced by Emil Artin in the 1920's to formalize topological objects that model the intertwining of several strings in the Euclidean 3-space.

In [1], Artin showed that braids with a fixed number n of strings form a group, called the *Artin braid group on n-strands*, denoted by $B(n)$. Since then, several important results have been proven, for example, the *Presentation Theorem*, which ensures the presentation of $B(n)$ in terms of generators and relations, and the *Representation Theorem* of this group as a subgroup of the group of automorphisms of the free group on n generators.

This work's main objective is to show that the subgroup of pure braids, braids with trivial permutation of the strings, is bi-orderable, i.e. will display a total ordering for $P(n)$ to be invariant under multiplication on both sides. The key to prove this is that free groups are bi-orderable and $P(n)$ is a semidirect product of free groups. The ordering will be defined using a combination of Artin's combing technique and Magnus expansion of free groups.

On the other hand, in [5] Rolfsen, Dynnikov, Dehornoy and Wiest, demonstrated topological reasons for the existence of a left-ordering of the braid groups over the disk, i.e., there is a strict total ordering of the braids that is invariant under multiplication from the left, but not right. So, it is natural to ask if there is a possible uniform ordering: a left-ordering of $B(n)$ which restricts to a bi-invariant ordering of $P(n)$. Perhaps surprisingly, the answer is that this is impossible.

In [3] we find the statement that $B(n)$ is a group as well as the theorems cited above. For the main result, we studied the article of D. J. Kim, and D. Rolfsen, [6], jointly with the Chapter XV of the book of P. Dehornoy, I. Dynnikov, D. Rolfsen, and B. Wiest, [5].

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Integral Equations as Generalized Ordinary Differential Equations and applications

Rafael Marques and Márcia Federson

We are interested in investigating qualitative properties of solutions of Volterra integral equations of second kind through the theory of generalized ordinary differential equations (generalized ODEs, for short). The reader may consult [2] and [3] for more information about integral equations.

J. Kurzweil and, independently, R. Henstock developed a concept of integral that is an extension of the Riemann integral, but equivalent to the non-absolute Perron integral. This integral originates a formal equation-like object defined by its solutions, known as generalized ODE. The book [4] is a detailed treatise of generalized ODEs.

In this work, we connect the theories of integral equations and generalized ODEs and apply the results to a population growth model, found in [1].

Acknowledgements: We would like to thanks FAPESP by financial support.

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Clock Operators and Foundations of Quantum Mechanics

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An ideal quantum clock is a physical system possessing an observable which provides an estimation of elapsed time from its measured values. The standard atomic clocks, widely used today, are not a truly quantum clock. They do not give the time but rather provide a known frequency so that the elapsed time follows from counting the number of periods of such frequency. This extra apparatus is not the subject of quantum mechanics as conventionally understood. Here, we will analyze the possibility of *Clock Operators*: the preparation of a system in a known state and the subsequent measure of a suitable observable provides an indication of the elapsed time, [4]. In this analysis we face a trouble, Pauli's Theorem. Wolfgang Pauli realized that a self-adjoint operator T canonically conjugated to the Hamiltonian operator H does not exist whenever the spectrum of H is bounded from below. Pauliâs proof was not rigorous, and, to be fair, he never claimed it to be so. However, it took some time for a rigorous mathematical formulation of the problem appears in literature, [6]. The refined version of Pauli's Theorem can be interpreted as follows: if we want to work with time observables, we cannot use the formalism of positive valued-measure, PVMs. Some attention has been given to the possibility of weakening the self-adjointness condition for the clock observables. This can be done by making use of periodic time covariant Positive Operators Valued Measures (POVMs) as clock observables based in [4]. We will briefly discuss the Alkeizer and Glazman result, [1] and the "Spectral Theorem to POVMs" using these results and standard functional analysis, [5], and a little change in the postulate of measure of quantum mechanics ([2], [3]).

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Spectral Sequences and Some Aplications

Wellington Marques de Souza and Victor Hugo Jorge Pérez

Spectral sequences were created by Jean Leray in a concentration camp during World War II motivated by problems of Algebraic Topology . At first, it appears as a tool to assist in calculating the cohomology of a sheaf. However, Jean-Louis Koszul presents a purely algebraic formulation for these sequences, which basically consists in calculating a total of homology complex associated to a double complex. We will focus our work on the definitions and results that allow us to demonstrate known results of algebra, as "The Five Lemma" and "The Snake Lemma" using spectral sequences.

Acknowledgements: We would like to thanks CAPES by financial support.

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