FRACTIONAL DIFFERENTIAL EQUATIONS: A NOVEL STUDY IN BANACH SPACES

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The fractional calculus can be considered in many ways, a novel topic, once it is only during the last thirty years that it has been the subject of specialized conferences and treatises. Everything has begun with the important applications discovered in numerous diverse and widespread fields in science, engineering and finance, see [1, 2, 3, 4, 5] among others.

Hence, motivated by the huge success of this applications we start trying to answer unsolved questions of this theory. We want to deal with the abstract fractional Cauchy problems of order $\alpha \in (0,1)$, i.e.,

$$\begin{cases} cD_t^\alpha u(t) = Au(t) + f(t, u(t)), \ \ t > 0 \\ u(0) = u_0 \in X, \end{cases}$$

where X is a Banach space, $A : D(A) \subset X \to X$ is a sectorial operator, cD_t^α is Caputo's fractional derivative and $f : \mathbb{R}^+ \times X \to X$ is a suitable function.

In general, we answer questions that were not completely studied: for instance, we analyze the existence of local mild solutions for the problem, and its possible continuation to a maximal interval of existence and using some general comparison results in Banach spaces we guarantee the global uniqueness and existence of solution. The case of critical nonlinearities and corresponding regular mild solutions is also studied.

References

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