A STUDY ON THE EQUATION OF HÉNON

MARIBEL BRAVO QUISPE - EDERSON MOREIRA DOS SANTOS

We will make a quantitative and qualitative study of positive solutions for the following Dirichlet problem for the Hénon equation,

$$(P) \begin{cases} -\Delta u &= |x|^{\alpha}|u|^{p-2}u & in & B, \\ u &= 0 & on & \partial B, \end{cases}$$

where B is the unit open ball of $\mathbb{R}^N$ centered at zero and $\alpha > 0$. We will show that for $p \geq 2_{\alpha}^*$, where

$$2_{\alpha}^* = \begin{cases} \frac{2(N+\alpha)}{N-2}; & N > 2, \\ \infty; & N = 1, 2, \end{cases} \qquad 2^* = \begin{cases} \frac{2N}{N-2}; & N > 2, \\ \infty; & N = 1, 2, \end{cases}$$

the problem does not have nontrivial solutions. In counterpart, for $1 < p < 2_{\alpha}^*$ with $p \neq 1$, the existence of radial positive solutions will be guaranteed. Moreover, the uniqueness of a positive solution will be guaranteed as long as $1 < p < 2$. In addition we will present results on the existence of Ground State solutions when $2 < p < 2^*$ (critical exponent of Sobolev). In this interval we will show that any Ground state solution exhibits the Foliated Schwarz symmetry and, in case α is sufficiently large, we will show that the no Ground state is radially symmetric. To complete our study, we will present results on the existence of multiple positive solutions. In particular, we will show that (P) has $\left[\frac{N}{2}\right] - 1$ non radial solutions in case $N \geq 4$, $\alpha > N + 2$ and $p \in (2, 2(N-1)/(N-3))$.

REFERENCES

[1] Badiale, M. and Serra, E. *Multiplicity results for the supercritical Hénon equation*, Adv. Nonlinear Stud. 4 (4) 453-467, **2004**.

[2] Li, Y.Y. *Existence of many positive solutions of semilinear elliptic equations on annulus* Journal of Differential Equations 83 (2) 348-367, **1990**.

[3] Ni, W.M. *A nonlinear Dirichlet problem on the unit ball and its applications* Indiana Univ. Math. J. 31 (6) 801-807, **1982**

[4] Smets, D. and Willem, M. *Non radial ground state for the Hénon equation*, Commun. Contemp. Math. 4 (3) 467-480, **2002**.

[5] Smets, D. and Willem, M. *Partial symmetry and asymptotic behavior for some elliptic variational problems*, Calc. Var. Partial Diferential Equations 18 (1) 57-75, **2003**.

(Maribel) ICMC-USP
E-mail address: maribq@icmc.usp.br

(Ederson) ICMC-USP
E-mail address: ederson@icmc.usp.br