I Workshop de Teses e Dissertações em Matemática
ICMC - Instituto de Ciências Matemáticas e de Computação
USP - Universidade de São Paulo
December 1-2, 2011

BASIC RESULTS FOR FUNCTIONAL DIFFERENTIAL AND DYNAMIC EQUATIONS INVOLVING IMPULSES

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1 Introduction

Let r, $\sigma > 0$ be given numbers and $t_0 \in \mathbb{R}$. The theory of retarded functional differential equations (see e.g. [6]) is a branch of the theory of functional differential equations concerned with problems of the form

$$\begin{cases} x'(t) = f(x_t, t), \quad t \in [t_0, t_0 + \sigma], \\ x_{t_0} = \phi, \end{cases} \tag{1.1}$$

where $f : \Omega \times [t_0, t_0 + \sigma] \to \mathbb{R}^n$, $\Omega \subset C([-r, 0], \mathbb{R}^n)$ and x_t is given by $x_t(\theta) = x(t + \theta)$, $\theta \in [-r, 0]$, for every $t \in [t_0, t_0 + \sigma]$. The equivalent integral form is

$$\begin{cases} x(t) = x(t_0) + \int_{t_0}^{t} f(x_s, s)\, \mathrm{d}s, \quad t \in [t_0, t_0 + \sigma], \\ x_{t_0} = \phi, \end{cases} \tag{1.2}$$

where the integral can be considered, for instance, in the sense of Riemann, Lebesgue or Henstock-Kurzweil.

In this paper, we focus our attention on more general problems of the form

$$\begin{cases} x(t) = x(t_0) + \int_{t_0}^{t} f(x_s, s)\, \mathrm{d}g(s), \quad t \in [t_0, t_0 + \sigma], \\ x_{t_0} = \phi, \end{cases} \tag{1.3}$$

where the integral on the right-hand side is the Kurzweil-Stieltjes integral with respect to a nondecreasing function g. We call these equations *measure functional differential equations*. As explained in [4], equation (1.3) is equivalent to

$$\begin{cases} \mathrm{D}x = f(x_t, t)\mathrm{D}g, \quad t \in [t_0, t_0 + \sigma], \\ x_{t_0} = \phi, \end{cases} \tag{1.4}$$

where $\mathrm{D}x$ and $\mathrm{D}g$ denote the distributional derivatives of the functions x and g in the sense of L. Schwartz.

We show that functional dynamic equations on time scales represent a special case of measure functional differential equations. Using this correspondence, we obtain various results concerning the existence and uniqueness of solutions, continuous dependence, and periodic averaging for functional dynamic equations on time scales.

Also, we extend these results to functional dynamic equations with impulses (which were already investigated by various authors, see e.g. [1], [2], [3], [7]). For obtain this extension, we introduce impulsive measure functional differential equations and show how to transform them into measure functional differential equations without impulses. Then, we introduce impulsive functional dynamic equations on time scales and demonstrates how to convert them to impulsive measure functional differential equations. Thus, using these results, we relate impulsive functional dynamic equations and measure functional differential equations. We employ this correspondence to obtain theorems on the existence and uniqueness of solutions, continuous dependence, and periodic averaging for impulsive measure functional differential equations and impulsive dynamic equations on time scales.

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