

# THE ARTIN TYPE REPRESENTATION OF BRAID GROUP AND A INVARIANT OF LINKS

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From a group  $H$  and a element  $h \in H$ , we define a representation  $\rho : B_n \rightarrow \text{Aut}(H^{*n})$ , where  $B_n$  denotes the braid group on  $n$  strands, and  $H^{*n}$  denotes the free product of  $n$  copies of  $H$ . We call  $\rho$  the Artin type representation associated to the pair  $(H, h)$ . Here we study various aspects of such representations.

Firstly, we associate to each braid  $\beta$  a group  $\Gamma_{(H,h)}(\beta)$  and prove that the operator  $\Gamma_{(H,h)}$  determines a group invariant of oriented links. We then give a topological construction of the Artin type representations and of the link invariant  $\Gamma_{(H,h)}$ , and we prove that the Artin type representations are faithful if and only if  $h$  is nontrivial.

## 1 Mathematical Results

**Teorema 1.1.** *Let  $\beta$  be a braid, and let  $\widehat{\beta}$  denote the braid closure of  $\beta$ . Let  $X$  be a CW-complex with basepoint  $P_0$  and let  $\alpha$  be a loop in  $X$ . Then*

$$\pi_1(\Omega(\widehat{\beta}, X)) \simeq \Gamma_{(H,h)}(\beta) \tag{1.1}$$

where  $H = \pi_1(X, P_0)$  and  $h$  is the element of  $H$  represented by  $\alpha$ .

**Teorema 1.2.** *Let  $\rho : B_n \rightarrow \text{Aut}(H^{*n})$  be the Artin type representation of  $B_n$  associated to the pair  $(H, h)$  where  $H$  is a group and  $h \in H$ .*

- (1) *If  $h \neq e_H$  then  $\rho$  is faithful.*
- (2) *If  $h = e_H$  then  $\text{Ker}(\rho)$  is the pure braid group and  $B_n/\text{Ker}(\rho) \simeq \mathfrak{S}_n$ , the symmetric group, acts by permutation of the free factors of  $H^{*n}$  (respecting of the isomorphisms  $\{\phi_1, \dots, \phi_n\}$ ).*

## References

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