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A COMPLETE INVARIANT TO MORSE-BOTT SYSTEMS

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Ever since the fundamental ideas of Poincar in the qualitative theory of dynamical systems, there has been intense interest into the global behaviour of such systems. Many important ideas and examples have grown out of the basic theory including chaos, knotted trajectories, fractal dimension and equivalence theory. But until now a very difficult question is, when two dynamical systems are equivalent? To answer this question to particular cases we generally use complete topological invariant. In this work we shall present a complete topological invariant for dynamical systems on closed and orientable two-dimensional surfaces with a Morse-Bott first integral.

Let M will denote a smooth connected closed and orientable n-manifold. Let $f : M \to \mathbb{R}$ a Morse-Bott function, what means that f is a function whose the critical points are organized as nondegenerate smooth critical submanifolds. Here a critical submanifold of f is called nondegenerate if the Hessian of f is nondegenerate on normal planes to this submanifold.

Let X a flow on M that admits the function f as a first integral. We take a Morse theoretical approach by working with a Morse-Bott function f associated to a flow X on M and by considering the level sets associated to the regular values of f. We propose to study the changes in the topology that are forced on the level sets as we pass through critical sets of f. In this context the main difference between the Morse and Morse-Bott functions is that the singular points of X and the critical points of f can be different.

To each critical set S of a flow X defined on M a basic block is associated. A basic brick N for S is defined using a Morse-Bott function $f : V \to M$ associated to the flow. Let $f(S) = c$ and choose $\epsilon > 0$ small enough so that c is the only critical value in $(c - \epsilon, c + \epsilon)$ and S is the only critical set at level c. Define the basic brick as the component N of the pre-image $f^{-1}(c - \epsilon, c + \epsilon)$ which contains S. To each singular point of X which is not a critical point of f we associated a star in the corresponding level set.

The connect graph obtained by the bricks and the stars is called Bott graph. Similar graphs are discussed under other hypotheses by Bosinov in [1] and by Oshemkov [3].

The purpose of this talk is show that the Bott graph under basic conditions is a complete topological invariant for systems on closed and orientable two-dimensional surfaces with a Morse-Bott first integral. The graph carries informations about both the flow and the surface topology where the flow is defined. To proof the completeness we compared the Bott graph and the Neumann complex orbital described in [2].

References

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