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On an Asymptotic Behavior of Periodic Functional Differential Equations

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We are interested on the study of asymptotic behavior of functional differential equation

$$\dot{x}(t) = a(t)x(t) + b(t)x(t-\omega) \tag{0.1}$$

with constant delay $\omega > 0$ and real $n\omega$-periodic continuous functions $a(\cdot)$ e $b(\cdot)$. Let $\mathcal{C} = \mathcal{C}([-\omega, 0], \mathbb{R})$ the state space of (0.1). For all solution function $x(t)$, define $x_t \in \mathcal{C}$ by $x_t(\theta) = x(t+\theta)$. The asymptotic behavior of similar equation of (0.1) when the delay is integer multiple of the period, was studied for many people like Frasson & Verduyn Lunel [1, Sec. 5], Hale & Verduyn Lunel [3, Sec. 8.3], Philos & Purnaras [5].

1 Spectral Theory and Results

We use the Floquet Theory and operator spectral theory for the monodromy operator $\Pi : \mathcal{C} \to \mathcal{C}$, given by $\Pi\varphi = x_{n\omega}$ where x is the solution of (0.1) with initial condition $x_0 = \varphi$. The difficult of the problem is to find the monodromy operator using the steps method. General results of spectral theory of functional differential equation can be find in [1, Sec. 5]. Furthermore, using Dunford representation of the spectral projection P_μ of Π, where μ is an element of spectrum of μ, we have

$$P_\mu = Res_{z=\mu}(zI - \Pi)^{-}1 \tag{1.2}$$

if μ is a simple multiplier we obtain the following expression of spectral projection

Teorema 1.1. *Let $\varphi \in \mathcal{C}$, μ a simple multiplier of Π,*

$$P_\mu\varphi(\theta) = e_1^T \Omega_{-\omega}^{\theta}(1/z) \lim_{z\to\mu}(z-\mu)\left[\begin{pmatrix} 0 & I \\ z & 0 \end{pmatrix} - \Omega_{-\omega}^{0}(1/\mu)\right]^{-1}(\varphi(-\omega)e_n + G_{\varphi,\mu}(0)) \tag{1.3}$$

where $\Omega_t^s(1/z)$ is the fundamental matrix solution of ODE $\dot{X}(\theta) = M(\theta)X(\theta)$ where $M(\theta) = (a_{ij}(\theta))$ with $a_{i,i-1}(\theta) = b(i\omega+\theta)$, $i = 1, \ldots, n-1$, $a_{1n}(\theta) = b(\theta)/z$, $a_{ij}(\theta) = 0$ in the other case, and

$$G_{\varphi,z}(\theta) = \frac{1}{z}\varphi(\theta)e_1 - \frac{1}{z}\Omega_{-\omega}^{\theta}(1/z)\varphi(-\omega)e_1 + \int_{-\omega}^{\theta}\Omega_s^t(1/\mu)(\frac{1}{z}b(\omega+\theta)\varphi(s)e_n)ds.$$

The next result, which proof can be found in [1, Sec. 5], provide an asymptotic behavior estimative of the solution $x_t(s,\varphi)$ for every $\varphi \in \mathcal{C}$.

Teorema 1.2. *Let μ_j, $j = 1, 2, \ldots$, the nonzero multipliers of monodromy operator $\Pi(s)$ ordered by decreasing modulus, and $\varphi \in \mathcal{C}$. If λ is an arbitrary real number, then there are positive constants ϵ and N such that for $t \geq s$*

$$\left\| x_t(s,\varphi)) - \sum_{|\mu_n| > e^{\gamma\omega}} P_{\mu_n}(s)x_t(s,\varphi)\right\| \leq Ne^{(\gamma-\epsilon)(t-s)}\|\varphi\| \tag{1.4}$$

Therefore, if we can compute the residue of

$$\left[\begin{pmatrix} 0 & I \\ z & 0 \end{pmatrix} - \Omega_{-\omega}^{0}(1/\mu)\right]^{-1}$$

at μ, we can find an explicit formula of P_μ. Therefore, the existence of a simple multiplier μ_d that dominates the others allows as to give an explicit formula for the large time behaviour of solutions of (0.1).

References

[1] Miguel V. S. Frasson and Sjoerd M. Verduyn Lunel. Large time behaviour of linear functional differential equations. *Integral Equations Operator Theory*, 47(1):91–121, 2003.

[2] Gohberg I., Goldberg S., Kaashoek M. A. *Classes of linear operator*, Vol. I, volume 49 of *Operator Theory Advanced and Applications.*, Birkhuser Verlag (Basel), 1990

[3] Jack K. Hale and Sjoerd M. Verduyn Lunel. *Introduction to functional-differential equations*, volume 99 of *Applied Mathematical Sciences*. Springer-Verlag, New York, 1993.

[4] I.-G. E. Kordonis, N. T. Niyianni, and c. G. Philos. On the behavior of the solutions of scalar first order linear autonomous neutral delay differential equations. *Arch. Math. (Basel)*, 71(6):454–464, 1998.

[5] Ch. G. Philos and I. K. Purnaras. Periodic first order linear neutral delay differential equations. *Appl. Math. Comput.*, 117(2-3):203–222, 2001.